**Examination of and the characteristic period of oscillation**

Figure 1 shows results of three trajectories as part of a sequence to study how the number of steps at steady states varies as a function of diffusion coefficient. It’s clear that declines with increasing – which we already knew from the paper – but here we’ll proceed under the hypothesis that the steady state arrived at in each case is a Turing pattern.

|  |
| --- |
|  |
| **Figure 1**. 1-variable formulation for three different values of the diffusion coefficient. See parameters at the end of this document. |

To illustrate this approach, Fig. 2 shows the profile resulting at the endpoint of the trajectory shown in the left panel of Fig. 1. This endpoint shows some deterioration at facet corners, which seems to plague all of the 1-variable runs as they approach steady state. Putting that aside, however, we see the usual V-shape comprised of (in this case) - steps.

|  |
| --- |
|  |
| **Figure 2**. Profile corresponding to the case of Fig. 1. |

From this information we can obtain the average width of a step,

(1)

where “T” is for “Turing,” for each trajectory in the sequence. This definition rests on the idea that steps are analogous to Turing’s characteristic pattern – we’re saying “analogous” because in fact the positions of those steps evolve over time; what is stationary is the V-shape, and the average width of steps in that V-shape, once steady state has been reached.

Values of as a function of this series of simulations are displayed in Fig. 3.

|  |
| --- |
|  |
| **Figure 3**. Points: values of as a function of this series of simulations, according to Eq. 1. Line: Eq. 3, using best-fit parameters and . |

Returning to Turing’s analysis, we have the prediction that

(2)

where is a "characteristic period of oscillation" (see, e.g., <https://pubs.aip.org/aip/jcp/article-abstract/102/6/2551/481538/Dependence-of-Turing-pattern-wavelength-on?redirectedFrom=fulltext>). We don’t get a good fit using Eq. 2, but we do get a good fit if we add an offset,

(3)

Figure 3 also shows this line with parameters and obtained by a least-squares error optimization algorithm.

How should we interpret ? What happens to the system – independently of – on a time scale of ? To investigate this question, Fig. 4 displays quasiliquid thickness values over time of the “zero-dimensional” or “non-diffusion” analog to our sequence – that is, all parameters are the same as the sequence of points appearing in Fig. 3, except that .

|  |
| --- |
|  |
| **Figure 4**. Time dependence of and for a non-diffusing trajectory. |

The growth rate is such that a new layer of ice is added every , which is way too far from the periodicity we see in Fig. 3. Overtones of this periodicity would appear at an even lower periodicity (higher frequency), so no help there.

Parameters for the trajectory:

